MATH 1830

UNIT 2 DERIVATIVES

2.1 EXPONENTIAL AND LOGARITHMIC FUNCTION APPLICATIONS

Pre-Class:

- Take notes on the videos and readings (use the space below).
- Work and check problem #1 in the 2.1 NOTES section.
- Complete the 2.1 Pre-Class Quiz.

Introduction

Exponential functions occur frequently in science and business and are commonly used in compound interest applications.

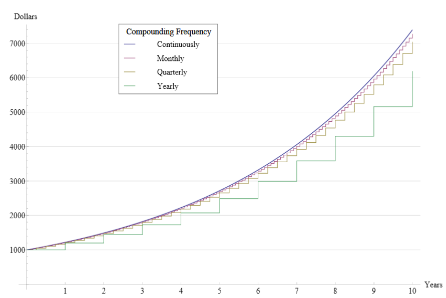
- The value of a $1000 investment returning 8% interest compounded monthly after 12 years would be calculated using the formula

  $$A = P\left(1 + \frac{r}{n}\right)^{nt},$$

  where:

  - A is the final amount in the account.
  - P is the principal.
  - r is the interest rate.
  - n is the number of compounding periods per year.
  - t is the number of years.

- The compounding frequency has a significant impact on the final amount of money (either saved or owed).



Compound Interest At Varying Frequencies

Starting with a principal of $1000, interest rises exponentially. Notice also that as time passes, a gap forms between the lines as less frequently-compounding methods increase at a lesser rate than more frequently-compounding methods.

Notes

Compounding Frequency

- Yearly:

$$A = 1000(1+\frac{.08}{1})^1 = 1080$$

- Quarterly:

$$A = 1000(1+\frac{.08}{4})^4 = 1082.43$$

- Monthly:

$$A = 1000(1+\frac{.08}{12})^{12} = 1083$$

- Daily:

$$A = 1000(1+\frac{.08}{365})^{365} = 1083.28$$

- Continuously (at every instant):

$$A = 1000 \cdot \lim_{n\to\infty} \left(1+\frac{.08}{n}\right)^n = 1083.29$$

Our focus will be on continuous compounding:

- What is e?
- Irrational number (similar to π)
- 2.718281828459.....
- Like π, e occurs frequently in natural phenomena
  - Growth of bacterial cultures
  - Decay of a radioactive substance
- Formal definition of e:

$$e = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$$

$$\approx \quad 2.718281829$$

Notes

Continuous Compounding Formula (appreciation and depreciation):

$$A = Pe^{rt}$$

CONTINUOUS COMPOUND INTEREST: Round all answers to two decimal places.

1. Hometown Bank offers a CD that earns 1.58% compounded continuously. If $10,000 is invested in this CD, how much will it be worth in 3 years?

2. Hometown Bank offers a CD that earns 1.58% compounded continuously. If $10,000 is invested in this CD, how long will it take the account to be worth $11,000?

3. Doubling Time: How long will it take money to double if it is invested at 5% compounded continuously?

4. Doubling Rate: At what nominal rate compounded continuously must money be invested to double in 8 years?

5. How long will it take money to triple if it is invested at 10.5% compounded continuously?

6. Radioactive Decay: A mathematical model for the decay of radioactive substances is given by

$$Q = Q_0 \ e^{rt}.$$

The continuous compound rate of decay of carbon-14 per year is $r = -0.0001238$. How long will it take a certain amount of carbon-14 to decay to half the original amount?

7. The estimated resale value R (in dollars) of a company car after t years is given by:

$$R(t) = 20000(0.86)^t.$$

What will be the resale value of the car after 2 years? How long will it take the car to depreciate to half the original value?

2.1 The Constant *e* and Natural Log Applications

Homework

Answer the following questions. Show all of your work. Round to two decimal places.

1. If you invested $1,000 in an account paying an annual percentage rate (quoted rate) of 2%, compounded continuously, how much would you have in your account at the end of

    a. 1 year
    b. 10 years
    c. 20 years
    d. 50 years

2. A $1,000 investment is made in a trust fund at an annual percentage rate of 12%, compounded continuously. How long will it take the investment to

    a. Double
    b. Triple

3. If $500 is invested in an account which offers 0.75%, compounded continuously find:

    a. The amount A in the account after t years.
    b. Determine how much is in the account after 5 years, 10 years, 30 years, and 35 years.
    c. Determine how long it will take for the initial investment to double.
    d. Find and interpret the average rate of change of the amount in the account from the end of the fourth year (t=4) to the end of the fifth year (t=5).

4. If $5000 is invested in an account which offers 2.125%, compounded continuously, find:

    a. The amount A in the account after t years.
    b. Determine how much is in the account after 5 years, 10 years, 30 years, and 35 years.
    c. Determine how long it will take for the initial investment to double.
    d. Find and interpret the average rate of change of the amount in the account from the end of the fourth year (t=4) to the end of the fifth year (t=5).

5. How much money needs to be invested now to obtain $5000 in 10 years if the interest rate in a CD is 2.25%, compounded continuously?

6. A mathematical model for depreciation of a car is given by $A = P(1-r)^t$, where A is defined as the value of the car after t years, P is defined as the original value of the car, and r is the rate of depreciation per year. The cost of a new car is $32,000. It depreciates at a rate of 15% per year. This means that it loses 15% of its value each year.

    a. Find the formula that gives the value of the car in terms of time.
    b. Find the value of the car when it is four years old.

7. A mathematical model for depreciation of an ATV (all-terrain vehicle) is given by $A = P(1-r)^t$, where A is defined as the value of the vehicle after t years, P is defined as the original value of the vehicle, and r is the rate of depreciation per year. The cost of a new ATV (all-terrain vehicle) is $7200. It depreciates at 18% per year.

   a. Find the formula that gives the value of the ATV in terms of time.
   b. Find the value of the ATV when it is ten years old.

8. Michigan's population is declining at a rate of 0.5% per year. In 2004, the state had a population of 10,112,620.

   a. Write a function to express this situation.
   b. If this rate continues, what will the population be in 2012?
   c. When will the population of Michigan reach 9,900,000?
   d. What was the population in the year 2000, according to this model?

https://sccmath.files.wordpress.com/2012/01/scc_open_source_intermediate_algebra.pdf

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UNIT 2 DERIVATIVES

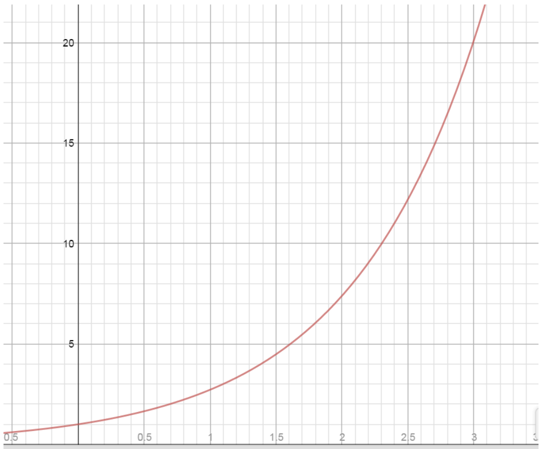
2.2 DERIVATIVES OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Pre-Class:

- Complete 2.1 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problems #1-4 in the 2.2 NOTES section.
- Complete the 2.2 Pre-Class Quiz.

Introduction

Finding the derivative of $f(x) = e^x$



x	f(x)=e^x
0	1
1	2.7183
2	7.3891
3	20.086
4	54.598
5	148.41
6	403.43

1. Calculate the slope of the secant line for each of the following intervals for the function $f(x) = e^x$.

    a. [1, 3]

    b. [1, 2]

    c. [1, 1.5]

2. What does the slope of the secant line represent?

3. Draw a tangent line at the point on the graph corresponding to x = 1 and calculate the slope.

4. What does the slope of the tangent line represent?

5. Compare the values of $f(1)$ and $f'(1)$. What do you notice?

Finding the derivative of $f(x) = lnx$

6. Try to find the derivative of $f(x) = lnx$ using the limit definition of the derivative, $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

7. Complete the table below to try to find the derivative of $f(x) = lnx$.

(Use your calculator and let h=0.00001 to represent h→0)

x	$\frac{\ln(x+h) - lnx}{h}$	$\lim_{h \to 0} \frac{\ln(x+h) - lnx}{h}$
1	$\frac{\ln(1+0.00001) - ln1}{0.00001}$	1
2		
3		
4		
5		

8. Based on your results what do you think the rule for the derivative of $f(x) = lnx$ is?

Notes

Derivatives of Exponentials and Logarithms

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} b^x = b^x \ln b \qquad (b > 0,\ b \neq 1)$$

$$\frac{d}{dx} \ln x = \frac{1}{x} \qquad (x > 0)$$

$$\frac{d}{dx} \log_b x = \left(\frac{1}{\ln b}\right)\left(\frac{1}{x}\right) \qquad (x > 0,\ b > 0,\ b \neq 1)$$

1. Find $f'(x)$ when $f(x) = 3x^3 + 4x^2 - 5x + 8$.

2. Find $f'(x)$ when $f(x) = \ln x - x^3 + 2x + e^x$.

3. Find $f'(x)$ when $f(x) = 4\ln x + 5e^x - 7x^2$.

4. Find $f'(x)$ when $f(x) = \ln x^8 - 3\ln x$.

Properties of Logarithms:

Use appropriate properties of logarithms to expand $f(x)$ and then find $f'(x)$.

5. $f(x) = 9 + 5\ \ln \frac{1}{x}$

6. $f(x) = x - 2\ \ln(5x)$

Tangent Lines:

Find the equation of the line tangent to the graph of f at the indicated value of x.

7. $f(x) = e^{x} + 2 \quad at \quad x = 0$

8. $f(x) = 1 + \ln x^{6} \quad at \quad x = e$

Applications:

9. The estimated resale value R (in dollars) of a company car after t years is given by

$$R\left(t\right) = 24000(0.84)^t$$

What is the instantaneous rate of depreciation (in dollars per year) after:

- 1 year?
- 2 years?
- 3 years?

2.2 Derivatives of Exponential and Logarithmic Functions

Homework

Find the derivative of each of the following.

1. $f(x) = e^x + \ln x - 2x^5 + 12$
2. $y = 7\ln x + 14x - \frac{1}{2}$
3. $g(x) = -5e^x - 12\ln x + 3x^3$
4. $f(x) = 8\sqrt{x} + 7e^x$
5. $y = \ln x^7 - 3\ln x$
6. $f(x) = 4\ln\frac{1}{x} + 8$
7. $y = e^x - 7\ln 5x + 14$
8. Find the equation of the line tangent to the graph of f at the indicated value of x.

   $f(x) = 2e^x - 1 \qquad at \quad x = 0$

9. **Find the equation of the line tangent to the graph of f at the indicated value of x.**

   $f(x) = 8\ln(x) \qquad at \quad x = e$

10. An editor of college textbooks has determined that the equation below models the sales of a calculus textbook, B (in thousands), based on the number of complimentary books sent to professors, x (also in thousands).

    $$B(x) = 3.24 + 1.6\ln\,(x)$$

    Find and interpret the instantaneous rate of change when 6000 complimentary books are sent to professors, x=6.

11. The percentage of mothers who returned to the work force within one year after they had a child for the years 1976 through 1998 can be modeled by

    $$P(t) = 36.025 + 6.27\ln(t)$$

    where t is years after 1977. (Source: Based on data from the Associated Press)

    a. What percentage of mothers returned to the work force within one year in 1998 and how rapidly was that percentage changing in 1998?
    b. On average, how rapidly did the percentage change from 1980 to 1990? c. What happens to the rate at which the percentage is growing as more years go by?

http://www2.fiu.edu/~rosentha/MAC2233/2233CE.htm (page 7 Section 3.3)

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UNIT 2 DERIVATIVES

2.3 DERIVATIVES OF PRODUCTS

Pre-Class:

- Complete 2.2 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problems #1-2 in the 2.3 NOTES section.
- Complete the 2.3 Pre-Class Quiz.

Introduction

1. The manager of a miniature golf course is planning to raise the ticket price per game. At the current price of $6.50, an average of 81 rounds is played each day. The manager's research suggests that for every $0.50 increase in price, an average of four fewer games will be played each day. Based on this information, find the function that represents revenue from rounds of mini golf, where n represents the number of $0.50 increases in ticket price.

    a. What must you do with this revenue function in order to find the rate of change?

    b. Find the rate of change for this revenue function when the manager increases the price of a round of mini golf by $1.50.

2. Find the rate of change for the function $y = (x^2 + 1)(x^2 - 2x + 1)$.

Notes

Derivatives of Products

THE PRODUCT RULE

If $y = f(x) \cdot g(x)$,

then $y' = f'(x) \cdot g(x) \ + \ \ f(x) \cdot g'(x)$.

Two Methods for Finding the Derivative:

Find the derivative two different ways.

- Simplify first and use the power rule.
- Use the product rule.

1. $m\left(x\right) = 2x^3\ \left(x^5 - 2\right)$

    a.

    b.

Find the derivative using the Product Rule.

2. $n(x) = 7x^2 \left(2x^3 + 5\right)$

3. $h\left(x\right) = 4x^3\ e^x$

4. $s\left(x\right) = 2x^5 \ln x$

5. $v(x) = (8x+1)\left(3x^2 \ -7\right)$

Tangent Lines

6. $r(x) = (5 - 4x)(1 + 3x)$

a. Find $r'(x)$.

b. Find the equation of the line tangent to the graph of r at $x = 2$.

c. Find the values of x where $r'(x) = 0$.

Derivatives with Radicals

7. Find y' for $y = \sqrt{x}\,\left(x^2 + 3x - 1\right)$.

8. Find $\frac{dy}{dx}$ for $y = \sqrt[3]{x}\,\left(x^6 + x^3\right)$.

Applications

9. Calculators are sold to students for 100 dollars each. Three hundred students are willing to buy them at that price. For every 5 dollar decrease in price, there are 30 more students willing to buy the calculator. The revenue function is given by the formula $R(d) = (100 - 5d)(300 + 30d)$.

   a. Find $R'(d)$.

   b. Find $R\left(3\right)$ and $R'\left(3\right)$. Write a brief interpretation of these results.

   c. Use the results above to estimate the total revenue after four $5 reductions in price.

2.3 Derivatives of Products

Homework

Find the derivative of each of the following.

1. $p(x) = (6x - 1)(5x + 2)$

2. $p(t) = 2t^2(t^3 + 4t)$

3. $h(z) = 3z^2 e^z$

4. $p(x) = 2e^x(x^2 - 3x + 5)$

5. $y = -x^2 \ln x$

6. $r(x) = (x^3 + x) \ln x$

7. $k(x) = (2x - 5)(x^2 + 1)$

8. $p(a) = (a^2 - 2a + 7)(2a^2 - a + 1)$

9. $p(t) = \sqrt{t}\,(t^2 + 5t - 2)$

10. $q(x) = (\sqrt{x} + 1)(\sqrt{x} - 3)$

11. $y = x^{-4}(x^2 - 7x + 1)$

12. $y = \frac{2}{x^2}\left(3x^4 - 6x + 2\right)$

13. If $z(x) = x^2(x^3 + 1)$, find the slope of that tangent line at $x = 1$.

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UNIT 2 DERIVATIVES

2.4 DERIVATIVES OF QUOTIENTS

Pre-Class:

- Complete 2.3 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problem #1 in the 2.4 NOTES section.
- Complete the 2.4 Pre-Class Quiz.

Introduction

1. The cost of manufacturing x MP3 players per day is represented by the function

$$C(x) = 0.01x^2 + 42x + 300 \quad 0 \leq x \leq 300.$$

   a. Determine the average cost function.

   b. Determine the marginal average cost function. What did you have to do to the average cost function in order to find the marginal average cost function?

2. Suppose the function $V(t) = \frac{50{,}000+6t}{1+0.4t}$ represents the value, in dollars, of a new car t years after it is purchased. Determine the rate of change in the value of the car.

Notes

Derivatives of Quotients

Rewriting a Function as a Quotient

THE QUOTIENT RULE

If $y = \frac{f(x)}{g(x)}$,

then $y' = \frac{f'(x)\ g(x)\ -\ f(x)\ g'(x)}{[g(x)]^2}$.

Two Methods for Finding the Derivative:

Find the derivative two different ways.

a. Simplify first and use the power rule.
b. Use the quotient rule.

1. $r(x) = \frac{x^5+4}{x^2}$

a.

b.

Find the Derivative of each Function using the Quotient Rule.

2. $b(x) = \frac{4x}{3x+8}$

3. $c(x) = \frac{x^2 - 9}{x^2 + 1}$

4. $h\left(x\right)=\ \frac{1+e^{x}}{1-e^{x}}$

5. $j\left(x\right)=\ \frac{3x}{4+\ln x}$

6. Find $\frac{dy}{dw}$ for $y = \frac{2w^4 - w^3}{6w-1}$

7. Explain how $f'(x)$ can be found without using the quotient rule: $f(x) = \frac{4}{x^3}$.

Tangent Lines

8. $h(x) = \frac{3x-7}{2x-1}$

a. Find $h'(x)$.

b. Find the equation of the line tangent to the graph of h at $x = 2$.

c. Find the values of x where h'(x) = 0.

Derivatives with Radicals

9. Find y' for $y = \frac{6\sqrt[3]{x}}{2x^2 - 5x + 1}$.

10. Find $\frac{dy}{dx}$ for $y = \frac{2x^2 - 2x + 3}{\sqrt[4]{x}}$.

Applications

11. A cable company has installed a new television system in a city. The total number N (in thousands) of subscribers t months after the installation of the system is given by $N\left(t\right) = \frac{178t}{t+5}$.

a. Find $N'(t)$.

b. Find $N\left(12\right)$ and $N'\left(12\right)$. Write a brief interpretation of these results.

c. Use the results above to estimate the total number of subscribers after 13 months.

12. According to economic theory, the supply x of a quantity in a free market increases as the price p increases. Suppose the number x of baseball gloves a retail chain is willing to sell per week at a price of $p is given by

$$x = \frac{100p}{0.1p+1} \qquad 30.00 \le p \le 190.00.$$

a. Find $\frac{dx}{dp}$.

b. Find the supply and the instantaneous rate of change (IRC) of supply with respect to price when the price is \$40. Write a brief verbal interpretation of these results.

c. Use the results above to estimate the supply if the price is increased to \$41.

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UNIT 2 DERIVATIVES

2.5 THE CHAIN RULE

Pre-Class 1.5A:

- Complete 2.4 homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problems #1-4 in the 2.5 NOTES section.
- Complete the 2.5A Pre-Class Quiz

Pre-Class 2.5B:

- Complete 1.5A Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- 1.5B Work and check problems #13 in the 2.5 NOTES section.
- Complete 2.5B Pre-Class Quiz

Introduction

1. The gas tank of a parked pickup truck develops a leak. The amount of gas, in liters, remaining in the tank after t hours is represented by the function $V(t) = 90\left(1 - \frac{t}{18}\right)^2 \quad 0 \leq t \leq 18$. How fast is the gas leaking from the tank after 12 hours?

2. Andrew and David are training to run a marathon. They both go on a run on Sunday mornings at precisely 7 A.M. Andrew's house is 22 km south of David's. One Sunday morning, Andrew leaves his house and runs west at 7 km/hr. The distance between the two runners can be modeled by the function

$$s(t) = \sqrt{130t^2 - 396t + 484},$$

where s is in kilometers and t is in hours. Determine the rate at which the distance between the two runners is changing.

Notes

General Derivative Rules Using the Chain Rule

$\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} \cdot f'(x)$

$\frac{d}{dx}\ln[f(x)] = \frac{1}{f(x)} \cdot f'(x)$

$\frac{d}{dx}e^{f(x)} = e^{f(x)} \cdot f'(x)$

Fill in the blank with an expression that will make the indicated equation valid. Then simplify.

1. $\frac{d}{dx}\ (3-7x)^6 = 6(3-7x)^5$ _____

2. $\frac{d}{dx}\ e^{5x-3} = e^{5x-3}$ ______

3. $\frac{d}{dx}\ ln\left(x^2 - x^4\right) = \frac{1}{x^2-x^4}$ ______

Find $f'(x)$ and simplify.

4. $f(x) = \left(8x^2 - 7\right)^5$

5. $f(x) = e^{3x^2 + 2x + 5}$

6. $f(x) = 2\ln\left(9x^2 - 5x + 21\right)$

7. $f(x) = (4x - 5\ln x)^7$

Horizontal Tangents

Finding the Equation of the Tangent Line, at $x = a$:

a. Find the y value by calculating $f(a)$: $(a, f(a))$.
b. Find the slope of the tangent line by calculating $f'(a)$: $m_{tan} = f'(a)$.
c. Equation of the tangent line: $y - f(a) = f'(a)\,(x - a)$.

Finding the Value(s) where the Tangent Line is Horizontal:

a. Set $f'(x) = 0$.
b. Solve for x.
c. Verify that each x is in the domain of $f(x)$ and $f'(x)$.

Find $f'(x)$ and simplify. Then find the equation of the tangent line to the graph of $f(x)$ at the given value of x. Find the values of x where the tangent line is horizontal.

8. $f(x) = (3x+13)^{1/2}$ at $x = 4$

Horizontal Tangent

9. $f(x) = 3e^{2x^2 + 5x - 4}$ $x = 0$

Horizontal Tangents:

10. $f(x) = \ln\left(1 - x^2 + 2x^4\right)$ at $x = 1$

Horizontal Tangent

Set each factor equal to zero.

Find the indicated derivative and simplify.

11. $\frac{d}{dt}\ 3\left(2t^4\ +t^2\ \right)^{-5}$

12. $\frac{dh}{dw}$ if $h\left(w\right) = \sqrt[5]{8w-1}$

13. $h'\left(x\right)$ if $h\left(x\right) = \frac{e^{4x}}{x^3\ +9x}$

14. $\frac{d}{dx}\ \left[x^5\ \ ln\left(3+x^5\ \right)\right]$

15. $G'(t)$ if $G(t) = \left(t - e^{9t}\right)^2$

16. y' if $y = \left[ln\left(x^2 + 7\right)\right]^{4/5}$

17. $\frac{d}{dw}\ \frac{1}{(w^2 - 5)^3}$

Horizontal Tangents

Find $f'(x)$ and simplify. Then find the equation of the tangent line to the graph of $f(x)$ at the given value of x. Find the values of x where the tangent line is horizontal.

18. $f(x) = x^2\ (3-2x)^4 \qquad x = 1$

Horizontal Tangent

19. $f\left(x\right)=\ \frac{x^4}{\left(2x-5\right)^2}$ $\qquad x=2$

Horizontal Tangent

20. $f(x) = e^{\sqrt{x}}$ when $x = 1$

Horizontal tangent

21. $f(x) = \sqrt{x^2 + 4x + 5}$ at $x = 0$

Horizontal tangent

Applications

22. COST FUNCTION: The total cost (in hundreds of dollars) of producing x pairs of sandals per week is: $C(x) = 6 + \sqrt{3x + 25}$ when $0 \le x \le 30$.

    a. Find $C'(x)$.

    b. Find $C'(17)$ and $C'(26)$. Interpret the results.

23. PRICE DEMAND EQUATION: The number of large pumpkin spice drinks (x) people are willing to buy per week from a local coffee shop at a price of p (in dollars) is given by:

$$x = 1000 - 60(p + 25)^{1/2}$$

when $3.50 \le p \le 6.25$.

    a. Find $\frac{dx}{dp}$.

    b. Find the demand and the instantaneous rate of change of demand with respect to price when the price is $4.50. Write a brief interpretation of these results.

24. BIOLOGY: A yeast culture at room temperature (68°F) is placed in a refrigerator set at a constant temperature of 38°F. After t hours, the temperature, T, of the culture is given approximately by

$$T = 25e^{-0.62t} + 38 \quad 0 \leq t \leq 4.$$

What is the rate of change of temperature of the culture at the end of 1 hour? At the end of 4 hours?

2.5A The Chain Rule

Homework

Find the derivative of each of the following.

1. $p(t) = (t^3 + 4)^5$
2. $f(x) = \sqrt{x^2 - 144}$
3. $g(x) = e^{2x^2 - 5x + 4}$
4. $p(a) = \ln\,(a^4 + 4a)$
5. $y = \frac{1}{\sqrt[3]{x - x^3}}$
6. $g(x) = 9(2x^2 + x - 7)^{-3}$
7. $y = (7 - 5\ln x)^3$
8. $q(t) = \ln(\ln\;5t)$
9. Find the equation for the tangent line to the curve $y = \sqrt{e^x + 8}$ at the point where $x = 0$.
10. The concentration of toxic material in a lake is related to the number of months that an manufacturing plant has been operating near the lake. This concentration of toxic material can be modeled by $A(t) = \left(0.7t^{1/4} + 5\right)^3$ where A is measured in parts per million (ppm).
    a. Find the model for the rate of change in the concentration of the toxic material in the lake.
    b. Find $A(20)$ and $A'(20)$and interpret the results.
    c. Use the results from part b to estimate the total amount of toxic material in the lake at 21 months.

2.5B The Chain Rule

Homework

Find the derivative of each of the following.

1. $p(t) = t^2(5t+1)^3$

2. $r(x) = (2x^2-3)(7x+4)^3$

3. $p(a) = a^3 \ln(a^5)$

4. $y = \frac{e^{x^2+x}}{4x-7}$

5. $h(x) = (6x+5)(x^2+4x+8)^{-2}$

6. $y = \frac{\ln{(9x+2)^2}}{x}$

7. $q(x) = \frac{e^{2x}}{e^{3x}+1}$

8. $h(x) = \frac{2\sqrt{x}}{(x^2-36)^3}$

9. Find an equation for the tangent line to the curve $n(x) = x^2 \ln x$ at the point where $x = e$.

10. The number of people in Knoxville who contract the flu can be modeled by $P(t) = \frac{15{,}000}{50e^{-0.3t}+1}$, where P is the number of people who contract the flu and t is the number of days after the outbreak began.

    a. Find the model for the rate of change of the number of people with the flu.

    b. Find $P(4)$ and $P'(4)$and interpret the results.

    c. Use the results from part b to estimate the total number of people with the flu in Knoxville after 5 days.

Unit 2 In-Class Review Problems

Find the derivative of each function. Show all of your work and simplify your answer.

1. $h(x) = \left(\frac{5}{x^2} - 3\right)\left(6x^2 + 1\right)$

2. $y = \sqrt[5]{(7x - 8)^3}$

3. $k(n) = \frac{4n-7}{\left(2n^4+5\right)^2}$

4. $f(x) = \ln\left(5x^2 + 3x\right)$

5. $r(w) = \left(w^2 - 2\right)e^{3w^2-7}$